Analysis of New Keynesian DSGE and Marshallian firm theory

Minseong Kim

Abstract. This writing - a “working paper” - is from the full writing that will soon appear in the full form. One part intends to talk about recent debates regarding New Keynesian DSGE models. Other parts are separate ones, tangentially related to New Keynesian DSGE, that are included for the purpose of fostering further discussion. The appendix parts discuss issues related to perfect competition or Marshallian firm theory. The final form of this paper will be much different from the current form, although contents will not change. Some appendix contents are missing at this point, especially some full log-linearization proofs. The corresponding part to this paper will discuss new medium-sized New Keynesian models and estimation issues with New Keynesian (NK) models and NK Phillips Curve.

1. Before Reading

In this version of writing, page 5 to page 12 basically describes the basic New Keynesian DSGE model. Anyone familiar with it should skip reading these pages. Currently, some appendix and sections are missing, and I will fill over these sections in the future version. Also, note that there is the corresponding part to this chapter, again about New Keynesian DSGE models. These writings will appear soon.

2. Lucas Critique

The last few chapters discussed mostly mainstream microeconomic model and “microeconomics-derived” macroeconomic models, such as general equilibrium. This chapter intends to discuss mainstream macroeconomic models. As we saw in the last chapters, microeconomics plays a vital role in mainstream macroeconomics. One may then ask why pure general equilibrium model is not used in macroeconomics. This is because of complexity underlying pure general equilibrium, as we cannot measure utility functions of every individual. Furthermore, there are some areas, such as the effects of monetary policy, that can be better described by simplified macroeconomics models. But before continuing on to discuss New Keynesian models that are
often used throughout the world, I would like to re-emphasize connections between microeconomics and macroeconomics in mainstream economics. I will only discuss briefly about real business cycle models. As with other chapters, I will use neoclassical and mainstream synonymously. History of modern macroeconomics, especially last 30 years, cannot be explained without Lucas critique, and therefore this critique deserves its place here. What Lucas critique says is the following:

Critique 1. *Macroeconomic activities must be modelled based on relationships and variables that are policy-invariant. The best optimal policy drawn from the studies of economy under one particular policy may not turn out to be the best optimal policy when the policy itself changes to the best optimal policy.*

Critique 2. *Keynesian macroeconomic models before Rational Expectation Revolution are often based on relationships and variables that are not policy-invariant. The best example is the use of original Phillips curve that statistically shows trade-offs between inflation and unemployment. When government begins to utilize Phillips curve relationship, agents react to the policy change, breaking down the original relationship.*

Critique 3. *Every macroeconomic models must have microfoundations, and any model without proper microfoundations is invalid.*

These three critiques are very powerful, in sense that this gives a guideline for macroeconomic models. The answer to this Lucas critique was met in New Classical economics by Finn E. Kydland and Edward C. Prescott: real business cycle (RBC) model, the first dynamic stochastic general equilibrium model (DSGE) model. However, it is important to note that DSGE models, which include RBC models, utilize representative agents. These models often work by one fictional household person representing entire households, one fictional firm representing entire firms and so on. The question then is, is this method consistent under neoclassical axiomatic doctrines mentioned in the previous chapters? This answer has generally been "approximately, yes" in mainstream economics, and "no at all" in heterodox economics. This diverging answer is due to the interpretation of Sonnenschein-Mantel-Debreu theorem. I leave further discussions of Sonnenschein-Mantel-Debreu theorem to the previous sections, but I will briefly again talk about the theorem. The theorem says that movements of equilibrium may not be described by single utility function that respects individual utility function restrictions and represents each agent group when
rational individual behaviour axioms are assumed. (As we by far all know, the direct consequence of this theorem is that there might be more than one equilibrium in macroeconomic world.) Mainstream economists interpret this as only general cases that can be solved when certain assumptions are adopted, such as gross substitution. Heterodox economists disagree, stating that gross substitution properties are not usually satisfied in reality - examples of financial assets and money are often used, but not limited to these - and different assumptions are yet to be completely shown by mainstream economics. To recall what gross substitution is, it says that when price of one product is increased, given that price remains the same for other products, excess demands for other products are increased or remain as before.

To understand some reasons why many heterodox economists disagree with accepting gross substitution axiom, consider spillover effects. If a firm is selling accessories to one product of the other firm and that other firm increases price, then as excess demand of the other firm decreases, there is possibility that the firm’s excess demand would also decrease. One may be mistaken in this issue, saying that multiple equilibria that often exists in New Keynesian DSGE models are reduced to one equilibrium by log-linearization and setting monetary policy rules - but this is actually irrelevant to the discussion above. Also, forward-looking behaviour that exists in DSGE and mostly not in general equilibrium is also irrelevant to the discussion above, as what we are discussing is whether economy can be modelled as single utility function for each sector at each time. That is, the issue underlined above stands by itself.

However, gross substitution itself does not also guarantee that entire economic system behaves as if there is one utility function. (Gross substitution axiom only guarantees that uniqueness of equilibrium can be attained.) To get the sufficient condition exactly, one most famous assumption is homothetic preference, which says that everyone’s preference is essentially similar. It should be noted, though, that many mainstream economists do note the implications of DSGE models and sometimes state that DSGE models are to be used only in specific cases. In this essence, whether Lucas critique has actually been incorporated into DSGE models still remains, though it is most objective to say that DSGE representative agent resolution to Lucas critique have been accepted by many mainstream economists.

One final word on this is that applications of Lucas critique depend on right microeconomic relationship. For example, for many Marxists,
connections with labour values, whether price or internal value, would be microfoundation. For neoclassicals, it would be utility maximization and profit maximization of individuals and firms that approximately resemble Homo Economicus. Many economists agree that agents in economy possess future-looking/forwarding behaviours, although they differ on methods that should be used to investigate into future-forwarding behaviours. For notes, DSGE models often use discounted expected utility value, how this is constructed will be seen when discussing New Keynesian DSGE models. Philosophical, logical and mathematical issues of neoclassical economic philosophy are handled in the first few chapters, and I intend to stop our discussion of connection between microeconomics and macroeconomics here. (There are also issues with neoclassical/Solow growth model and production functions that underlie DSGE models, as seen before. For example, "technology" coefficient may actually not be measuring technology coefficients, but actually measuring wage-profit relationship, but one can just look at first and last few chapters.)

3. New Keynesian DSGE

After the demise of Old Keynesians, or MIT Keynesian due to whatever reasons we can attribute to, such as stagflations, political failures or whatever, New Classicals, or Freshwater economists, were victorious - though let us not forget that there were Post-Keynesians, including Cambridge Keynesians, that thought MIT Keynesians are not Keynesians at all and many MIT Keynesians are taking Phillips curve too seriously. But this victory did not last so long. Many economists soon questioned the empirical fit of RBC and began to create different types of rational expectation models based on RBC. One branch of them became quite eminent and these people focused on incorporating monopolistic competition and price stickiness into RBC models. These people became known as New Keynesians. I now show the baseline New Keynesian models below. One important fact about baseline New Keynesian models is: **There is no investment and government in this model.** Therefore, \( y_t = c_t \) or \( Y_t = C_t \).

Households

\[
\max_{C_t, B_t, N_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)
\]

\[
U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}
\]
$U(C_t, N_t)$ describes utility of consumption $C_t$ and working $N_t$ amount at time $t$, and the household representative agent maximizes discounted value of sum of expected utility to infinite time horizon. $\beta$ is a unique intertemporal discount factor. Utility function is modelled after constant-relative-risk-averse (CRRA) utility functions. Derivations of this form of utility function can be done by referencing risk aversion literature. Note that this when letters that variables are capitalized, this means that variables have not been log-linearized. Greek letters are all assumed not to have been log-linearized. We define $\rho = -\log \beta$.

By Dixit-Stiglitz monopolistic competition aggregation model,

$$C_t = \left( \int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} \, di \right)^{\frac{1}{1-\epsilon}}$$

where $\epsilon$ represents intratemporal elasticity of substitution between differentiated goods, $i$ represents a single variety out of all varieties ($j$ therefore represents a different variety) and,

$$\epsilon = -\frac{\partial C_t(j)}{C_t(j)/P_t(i)} \frac{\partial P_t(i)}{P_t(i)}$$

Note that $\epsilon$ is constant in this model. (CES)

Each variety ranges from 0 to 1, represented as (real line segment) continuum $i \in [0, 1]$.

Budget constraint is:

$$\int_0^1 P_t(i) C_t(i) \, di + Q_t B_T \leq B_{t-1} + W_t N_t + J_t$$

$C_t(i)$ represents amount of consumption of good variety $i$ at time $t$, $P_t(i)$ represents the price of good variety $i$ at time $t$, $N_t$ represents hours of work at time $t$, $W_t$ represents nominal wage at time $t$, $B_t$ represents one-period bonds purchased at time $t$ at the price of $Q_t$ and $J_t$ represents dividends from ownership of firms or other lump-sum payments that add toward income. Solvency condition states that

$$\lim_{T \to \infty} E_t(Q_{t,T} + B_{t,T}) \geq 0$$

We define aggregate price index, as in the Dixit-Stiglitz/CES literature,

$$P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} \, di \right)^{\frac{1}{1-\epsilon}}$$

and inflation variable,

$$\Pi_{t+1} = \frac{P_{t+1}}{P_t}$$
And again from the Dixit-Stiglitz literature,

\[ C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \]

where \( \int_0^1 P_t(i) C_t(i) \, di = P_t C_t \) is satisfied.

By the reasons of solving optimization problem of the utility function using Lagrange multiplier method, at equilibrium,

\[ \frac{U_N}{U_C} = \frac{W_t}{P_t} = C_t^\sigma N_t^\varphi \]

Log-linearization results in:

\[ w_t - p_t = \sigma \, c_t + \varphi \, n_t \]

where \( U_A \) represents first-order partial derivative of the utility function of variable \( A \), and \( w_t \) and \( p_t \) is log-value of \( W_t \) and \( P_t \) (\( \log W_t = w_t \)), and from this point, all lower-case letters will share this trend, and by the same reasons,

\[ Q_t = \beta E_t \left( \frac{U_{C_{t+1}}}{U_C} \frac{P_t}{P_{t+1}} \right) = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \]

Note that

\[ i_t = -\log Q_t \]

by definition. From the equation that describes \( Q_t \), we first think of the case where inflation \( \Pi \) is constant and consumption growth \( \Gamma \) is constant. Then the equation of \( Q_t \), now \( Q \), would read:

\[ Q = \beta \frac{\Gamma^{-\sigma}}{\Pi} \]

Log-linearizing it,

\[ q = -\rho - \sigma \gamma - \pi \]

As \( i_t = -\log Q_t \) and as we are here assuming constant inflation and constant consumption growth,

\[ i = \rho + \pi + \sigma \gamma \]

Now we look at the original equation of \( Q_t \), without assumption of constant inflation and constant consumption growth. Divide the original equation of \( Q_t \) by \( Q_t \) and we get,

\[ 1 = \frac{\beta}{Q_t} E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \]
Log-linearize it using log-linearizing methods and first-order Taylor expansion (the exact method would be provided in the appendix) and we get approximately,

\[ 1 = 1 + i_t - \sigma E_t \Delta c_{t+1} - \pi_{t+1} - \rho \]

where \( \Delta c_{t+1} = c_{t+1} - c_t \).

From this log-linearized equation, we get consumption Euler equation that will be crucial for our analysis of New Keynesian DSGE.

**New Keynesian Consumption Euler Equation**

\[ c_t = E_t [c_{t+1}] - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}] - \rho) \]

or equivalently,

\[ y_t = E_t [y_{t+1}] - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}] - \rho) \]

, where \( Y_t \) is production amount, always equal to \( C_t \) in equilibrium and \( y_t \) is its log-value.

Now, interpretation of this Euler equation is very important. This equation, in ordinary language should be read as following: \( c_t \) is determined by \( E_t [c_{t+1}] \), \( i_t \), \( E_t [\pi_{t+1}] \) and \( \rho \). While it is mathematically true that \( E_t [c_{t+1}] \) determined by \( c_t \) and other variables (for note, \( \rho \) is constant variable), one should not use this interpretation. This is because when solving equilibrium, interpretation does matter and forward-solving and backward-solving become crucial. We will go over this in the later parts of this chapter.

**Firms**

Production function of the representative firm is described by

\[ Y_t = A_t N_t^{1-\alpha} \]

and log-linearized version,

\[ y_t = a_t + (1 - \alpha) n_t \]

where \( N_t \) is hours worked, \( A_t \) is the level of technology, evolving exogenously.

Price stickiness is assumed in the fashion of Calvo stickiness. At each period, random \( 1 - \theta \) fraction of firms adjust price, while remaining \( \theta \) fraction retains its price. When firms adjust price to properly maximize their profits, their price would be \( P^*_t \). Therefore, aggregate price level, by
referring to Dixit-Stiglitz literature, is equal to:

\[ \Pi_t = \left( \theta P_{t-1}^{\epsilon} + (1 - \theta)(P_t^*)^{\epsilon} \right)^{\frac{1}{\epsilon}} \]

Divide by \( P_{t-1}^{\epsilon} \) and we get

\[ \Pi_t^{1-\epsilon} = \theta + (1 - \theta) \left( \frac{P_t^*}{P_{t-1}} \right)^{\frac{1}{\epsilon}} \]

Log-linearizing around \( \Pi_t = 1 \) (further derivations will be provided in appendix and references),

\[ \pi_t = (1 - \theta)(p_t^* - p_{t-1}) \]

Unlike classical settings, as only \( 1 - \theta \) fraction of firms adjust price to the best profit level, profit-maximization equation for firms changing price at time \( t \) is (note that as in convention, we drop variety variable \( i \) for convenience. And this is anyway unnecessary for discussion here):

\[
\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[ Q_{t,t+k} \left( P_{t-k}^* Y_{t+k,t} - \Psi_{t+k} \left( Y_{t+k,t} \right) \right) \right]
\]

where \( \Psi_{t+k} \left( Y_{t+k,t} \right) \) is a cost function at time \( t + k \) with parameter \( Y_{t+k,t} \) that represents quantity firms produce at \( t + k \) given that they adjusted their price at time \( t \) only. So the whole equation is about maximizing sum of expected nominal profits given that firms adjust price at time \( t \) and probability of not adjusting price at each period is \( \theta \). Note that this equation has start from a new time \( t + e \) when firms adjust their price at time \( t + e \). This equation arises because firms that adjust their price are chosen randomly - therefore one has to calculate from time \( t \) to infinity, given stochastic discount value \( Q_{t,t+k} \). The stochastic discount value \( Q \) is chosen, because \( Q \) is related to interest rate \( i_t \) as seen in the household parts. Using information from the household optimization parts,

\[ Q_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)^\sigma \left( \frac{P_t}{P_{t+k}} \right) \]

By referencing to Dixit-Stiglitz literature shown above,

\[ Y_{t+k,t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \]

Then the profit-maximization equation becomes:

\[
\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right) \times \left[ P_t^* \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} - \Psi_{t+k} \left( \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \right) \right] \right\}
\]
We define $\psi$ to be marginal cost: $\psi = \Psi'(Y)$ and $M = \frac{\epsilon}{1-\epsilon}$
Then, first-order condition would be:

$$\sum_{k=0}^{\infty} \theta_k E_t \left[ Q_{t,t+k} Y_{t+k|t} (P^*_t - M\psi_t) \right] = 0$$

. When price rigidity $\theta$ is 0, first-order condition is simply:

$$\max_{P^*_t} P^*_t Y_t - \Psi_t (Y_t)$$

Note again that $\Psi_t (Y_t)$ is a total cost function $\Psi_t$ of variable $Y_t$, not $\Psi$ multiplied by $Y$. As we are assuming monopolistic competition, $P^*_t = M\psi_t$ and $M$ is frictionless gross markup over marginal cost. We can then define frictionless real marginal cost $\psi_t/P^*_t = MC = 1/M$ (case where there is no price rigidity). Note that we will use $MC_{t+k|t}$ for the case when price rigidity exists. If we divide (where price rigidity exists) first-order condition by $P_t - 1$, using $MC_{t+k|t} = \psi_{t+k|t}/P_{t+k}$, $Q_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right)$ and $Y_{t+k|t} = \left( \frac{P_t}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$, we get:

$$\sum_{k=0}^{\infty} (\beta\theta)^k \left( \frac{P^*_t}{P_{t+k}} \right)^{1-\epsilon} \frac{C_{t+k}^{1-\sigma}}{P_t^{1-\sigma}} = \sum_{k=0}^{\infty} (\beta\theta)^k \left( \frac{P^*_t}{P_{t+k}} \right)^{-\epsilon} \frac{C_{t+k}^{1-\sigma}}{P_t^{1-\sigma}} M C_{t+k|t} \Pi_{t-1,t+k}$$

Simplify, and

$$\sum_{k=0}^{\infty} (\beta\theta)^k \left( \frac{P^*_t}{P_{t+k}} \right)^{1-\epsilon} \frac{C_{t+k}^{1-\sigma}}{P_t^{1-\sigma}} = \sum_{k=0}^{\infty} (\beta\theta)^k \left( \frac{P^*_t}{P_{t+k}} \right)^{1-\sigma} M C_{t+k|t} \Pi_{t-1,t+k}$$

Now perform log-linearization at the zero-inflation steady state (At steady state, $P^*_t = P_{t+k}$ for any k) (Use $X \approx \bar{X}(1 + \hat{x})$):

$$\sum_{k=0}^{\infty} (\beta\theta)^k \bar{X} \left[ 1 + (p_t^* - p_{t-1}) + (1 - \sigma)\hat{c}_{t+k} - (1 - \epsilon)\hat{p}_{t+k} \right]$$

equals

$$\sum_{k=0}^{\infty} \bar{X} \left[ 1 + (1 - \sigma)\hat{c}_{t+k} - (1 - \epsilon)\hat{p}_{t+k} + \hat{m}c_{t+k|t} + \pi_{t-1,t+k} \right]$$

Equivalently,

$$\sum_{k=0}^{\infty} (\beta\theta)^k (p_t^* - p_{t-1}) = \sum_{k=0}^{\infty} \left( \hat{m}c_{t+k|t} + \pi_{t-1,t+k} \right)$$
(left-hand side converges to \((1/(1 - \beta \theta))(p_t^* - p_{t-1})\)) Therefore,

\[ p_t^* = \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \left[ mc_{t+k|t} + p_{t+k} \right] \]

Equivalently,

\[ p_t^* = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \left[ \hat{m}c_{t+k|t} + p_{t+k} \right] \]

Now we must specify labor market. As labor supply equals labor demand, the following holds:

\[ N_t = \int_0^1 \left( \frac{N_t(i)}{A_t} \right)^{\frac{1}{\alpha}} di \]

equals

\[ N_t = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{\alpha}} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{\frac{1}{\alpha}} \frac{1}{\alpha} di \]

Then approximately,

\[ (1 - \alpha) n_t = y_t - a_t + d_t \]

\[ d_t = (1 - \alpha) \log \int_0^1 (P_t(i)/P_t)^{-\epsilon/(1-\alpha)} di \]

is a measure of price dispersion across firms. Approximating \(d_t\) by first-order approximation (to be provided in appendix),

\[ y_t = a_t + (1 - \alpha)n_t \]

Real marginal costs:

\[ MPN_t = \frac{\partial Y_t}{\partial N_t} = (1 - \alpha) A_t N_t^{-\alpha} \]

Using equation \(y_t = a_t + (1 - \alpha)n_t\),

\[ mc_t = w_t - p_t - \frac{1}{1 - \alpha} (a_t - \alpha y_t) - \log (1 - \alpha) \]

For firms at time \(t + k\) that adjusted price only at \(t\), real marginal costs are:

\[ mc_{t+k|t} = w_{t+k} - p_{t+k} - \text{mp}_{t+k} \]

\[ mc_{t+k|t} = mc_{t+k} + \frac{\alpha}{1 - \alpha} (y_{t+k|t} - y_{t+k}) \]

Recall Dixit-Stiglitz literature.

\[ Y_{t+k|t} = \left( \frac{P^*_t}{P_{t+k}} Y_{t+k} \right) \]
Log-linearizing this equation gives:

\[ y_{t+k|t} = -\epsilon (p^*_t - p_{t+k}) + y_{t+k} \]

Then,

\[ mc_{t+k|t} = mc_{t+k} - \frac{\alpha \epsilon}{1 - \alpha} (p^*_t - p_{t+k}) \]

Refer to the equation of \( p^*_t \) and \( p_{t-1} \), and using \((1 - \beta \theta)^{\sum_{k=0}^{\infty} (\beta \theta)^k E_t[p_{t+k} - p_{t-1}] = \sum_{k=0}^{\infty} (\beta \theta)^k E_t[p_{t+k} - p_{t-1}]}\) we get

\[ p^*_t = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t[\Theta \hat{m}c_{t+k} + (p_{t+k} - p_{t-1})] \]

equals

\[ p^*_t - p_{t-1} = (1 - \beta \theta) \Theta \sum_{k=0}^{\infty} (\beta \theta)^k E_t[\hat{m}c_{t+k}] + \sum_{k=0}^{\infty} (\beta \theta)^k E_t[\pi_{t+k}] \]

where \( \Theta = (1 - \alpha)/(1 - \alpha + \alpha \epsilon) \) Some rearrangements and substitutions will result in

\[ p^*_t - p_{t-1} = \beta \theta E_t[p^*_t - p_t] + (1 - \beta \theta) \Theta \hat{m}c_t + \pi_t \]

Note that \( \pi_t = (1 - \theta) (p^*_t - p_{t-1}) \) Then we get:

**New Keynesian Phillips Curve**

\[ \pi_t = \beta E_t[\pi_{t+1}] + \lambda \hat{m}c_t \]

where \( \lambda \) is defined as \( \lambda = \frac{(1 - \theta)(1 - \beta \theta)}{\beta} \Theta \)

In the future, when discussing New Keynesian Phillips curve and consumption euler equation, we assume that some parameters are constant.

Recall that \( w_t - p_t = \sigma y_t + \varphi n_t \).

\[ mc_t = w_t - p_t - mpn_t \]

\[ mc_t = (\sigma y_t + \varphi n_t) - (y_t - n_t) - \log (1 - \alpha) \]

\[ mc_t = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log (1 - \alpha) \]

Natural level of output is denoted as \( y^n_t \), natural real interest rate as \( r^n_t \). \( y^n_t \) is equilibrium output under **flexible price**. Note \( mc = -\mu \).

\[ mc = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y^n_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log (1 - \alpha) \]
Equivalently,

\[ y^n_t = \psi^n_y a_t + f^n_y \]

where \( f^n_y \) and \( \psi^n_y \) are defined as:

\[ f^n_y = -\frac{(1 - \alpha)(\mu - \log(1 - \alpha))}{\sigma(1 - \alpha) + \varphi + \alpha} \]

\[ \psi^n_y = \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} \]

Subtracting \( mc \) from \( mc_t \) results in:

\[ \hat{mc}_t = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \]

where \( \hat{y}_t \) is defined as \( \hat{y}_t = y_t - y^n_t \). New Keynesian Phillips curve then can be arranged to be an output gap version:

\[ \pi_t = \beta E_t [\pi_{t+1}] + \kappa \hat{y}_t \]

where \( \kappa \) is defined as \( \kappa = \lambda \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \). By adding \( E_t [\delta y^n_{t+1}] \) to consumption euler equation, we get:

\[ \hat{y}_t = E_t \left[ \hat{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}] - r^n_t) \right] \]

where \( r^n_t = \rho + \sigma E_t [\delta y^n_{t+1}] = \rho + \sigma \phi^n_y E_t [\delta a_{t+1}] \)  

**Taylor rule/central bank** Taylor rules state that central bank set nominal interest by the following:

\[ i_t = \rho + \phi \pi_t + \phi_y \hat{y}_t + \nu_t \]

where \( \nu_t \) is monetary policy shock and deviation from strict Taylor rules.

Please note that central bank is required to limit the number of equilibrium - otherwise, the model is not closed. **Final log-linearized model** While huge chunks of inks were spent on the derivation of log-linearized model, for many analysis, only a few equations of log-linearized model are needed. The following is the equations used to estimate equilibrium and for analysis:

**Output gap consumption euler equation**

\[ \hat{y}_t = E_t \left[ \hat{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}] - r^n_t) \right] \]

**New Keynesian Phillips curve**

\[ \pi_t = \beta E_t [\pi_{t+1}] + \kappa \hat{y}_t \]
Taylor Rules

\[ i_t = \rho + \phi \pi_t + \phi_y \bar{y}_t + \nu_t \]

For solving equilibrium, Taylor rules are often substituted to output gap consumption euler equation, as we will see in solving equilibrium by Blanchard-Khan methods.

Solving equilibrium by Blanchard-Khan

This is to be filled in.

Effects of monetary policy shock through undetermined coefficients

This is to be filled in.

Potential conflicts of Calvo stickiness with Lucas Critique

As can be noticed above, Calvo stickiness just assumes that exogenous \( \theta \) fraction of firms chosen randomly does not adjust their price. While there is a potential issue with how this stickiness is incorporated into aggregate price index, for this time, let us ignore this issue and focus on a possible conflict with Lucas critique. Lucas critique says that every macroeconomic model should be modeled based on policy-invariant parameters. Is Calvo stickiness \( \theta \) policy-invariant? In some economic condition, firms may be more inclined to incorporate information into pricing. This may occur due to changes of fiscal or monetary policies. We cannot guarantee that Calvo stickiness is really policy-invariant, unless menu costs, which say that costs of changing price make firms not change their price despite the fact that the better optimal price exists, are completely accountable for price stickiness, and their effects on \( \theta \) remains constant. So far, this has not been established, and empirical tests of prevalence of menu costs in reality have been mixed in mainstream economics.

Consumption Euler equation paradox?

Several people, from John Cochrane (2011) to Nick Rowe have argued that New Keynesian DSGE models are faulty. One of the arguments made, especially by Nick Rowe, is that consumption euler equation makes policies often made by New Keynesians. Recall consumption euler equation:

\[ c_t = E_t [c_{t+1}] - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}] - \rho) \]

Suppose that central bank sets target real interest rate \( r_t = i_t - E_t [\pi_{t+1}] \). Let us say that \( c_t \) is known. For note, it is crucial to understand that \( c_t \) is set after \( i_t \) is set. Then if real interest rate was set high, consumption growth rate would be greater. As such, these people have claimed New Keynesian models behave weirdly. Note that \( C + I + G = Y \), \( I \) investment, \( G \) govern-
ment spending, by accounting. To simplify matter, we drop investment, and assume \( C + G = Y \). Now, \( \tilde{C}, \tilde{G}, \tilde{Y} \) are consumption, government spending and output gap from natural rate. By consumption euler equation, \( \tilde{C}' \) is a positive function of real interest rate. If we assume that output \( Y_t \) always is at natural rate, \( \tilde{Y}' = 0 \) always. Therefore, \( \tilde{G} \) is a negative function of real interest rate. This means that in order to raise real interest rate at natural level, rate of change of government spending gap from trend line needs to negative. (To simplify the word, while not really accurate, it is safe to generalize this as ”rate of change of government spending needs to decrease through time to increase natural real interest rate.” This implies that in order to escape zero interest rate bound, government needs to announce that government spending will decline throughout time. The question is, is this accurate analysis?

In perfect foresight rational expectation model, this logic is completely true, as expectation is self-fulfilled - \( E_t [c_{t+1}] = c_{t+1} \). But in New Keynesian models, central bank can ”fool” expectation and create monetary shocks, by raising or lowering interest rate than its expected interest rate given by Taylor rules. Note again that the shock persistence from \( t - 1 \) to \( t \) (some shocks at \( t - 1 \) carry over to time \( t \)) is already brought into expectation of an agent at time \( t \). By using the solution method of undetermined coefficient, it was shown that interest rate increase shock is contractionary. The whole monetary policy, in New Keynesian models, is based on monetary shocks and Taylor rules, and without them, the models being criticized are not New Keynesian.

But we can generalize New Keynesian models and get rid of Taylor rules. Now central bank directly controls real interest and inflation rate and does everything it can to maintain that same real interest and inflation rate at every period. Whether this is possible would not be considered and deferred until the second part of this chapter. Let us suitably assume that people realize that real interest and inflation rate will be always the same and anchor their real interest rate and inflation expectation to central bank’s real interest and inflation rate target. This implies that all variables connected to inflation and inflation expectation automatically adjust. Also assume that there is no external shock to demand, cost and so on. Then New Keynesian Phillips curve would read:

\[
\pi_t = \beta \pi_t + \kappa \tilde{y}_t
\]

As \( \beta \) must be less than 1 to avoid solution explosion, this would lead to \( (1 - \beta)\pi_t = \kappa \tilde{y}_t \). \( \kappa \) is constant, which means that \( \tilde{y}_t \) is solely determined by \( \Pi_t \) set by central bank. If \( \alpha \) is less than 1, \( \tilde{y}_t \) is always positive, which is a nonsense, as \( \tilde{y} \) must be in average 0 - otherwise, natural rate hypothesis
strictly does not hold - that classical theory is always right in long-run. A similar point has indeed been raised in McCallum (1998), which said that if central bank succeeds on creating expectation that is disinflationary over time, $\tilde{y} > 0$ will be permanent, violating strict natural rate hypothesis and our intuition that deflation is a bad thing for economy. For this McCallum critique, however, there is potential defence that states that if central bank strictly follows Taylor rules, space for this possibility is very small. But the case of inflation expectation and present inflation being anchored, as in above, is highly relevant to reality. (for some people and economists, stabilizing inflation rate so that it is similar to being kept equal is the reason for the existence of central bank.) Recall consumption euler equation:

$$c_t = E_t [c_{t+1} - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}] - \rho)$$

As $\rho$ is constant and inflation rate are assumed to be constant, higher preset constant nominal interest rate is, higher consumption (therefore output) growth is. But we’ve seen that if inflation rate is constant, $\tilde{y}$ is purely determined by inflation rate. But as $\tilde{y}$ is fixed by New Keynesian Phillips curve (NKPC), central bank cannot take advantage of higher interest rate causing higher growth rate. If not, basic New Keynesian model fails. To summarize, if central bank can maintain same inflation and real interest rate (thereby same nominal interest rate) and keep expectations anchored to targets, basic New Keynesian model without Taylor rule may fail to have an equilibrium if interest rate is not set properly, but this scenario is ruled out as non-equilibrium behaviour. To specify what real interest rate would be, the set of nominal and real interest rate that guarantees formation of equilibrium at pre-set inflation rate is natural real rate of interest. This can be seen at output gap version of consumption euler equation:

$$\tilde{y}_t = E_t [\tilde{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}] - r^n_t)$$

where $r^n_t$ is natural rate of interest. Then output gap would forever be greater than 0 given that $\alpha$ is less than 1. Taylor rules are not and do not have to be used, as we assumed that expected and present inflation rate are equal. Another more reality-relevant scenario would be when potential output matches with real output. I will show that at natural rate of output, New Keynesian models exhibit hyperinflation if not struck down by external or monetary shocks. (Note that monetary shocks are the ones that are outside the Taylor rules.) Suppose that output gap is zero every time. Then New Keynesian Phillips curve would be:

$$\pi_t = \beta E_t [\pi_{t+1}]$$
As \( 0 < \beta < 1 \), \( \pi_t < E_t [\pi_{t+1}] \). As we are working with rational expectation models, without any monetary, demand or any type of shock, expectation is fulfilled. Therefore, assuming no shocks, \( \pi_t < \pi_{t+1} \) when output gap is zero. This is similar to hyperinflation scenario, and while this mathematically is not problematic, hyperinflation when output gap is zero is not really a good consequence of a model. Again, this involves opinions, so it is futile to continue on with value judgement. I would just like to restate the fact that basic New Keynesian DSGE model has hyperinflation equilibrium when output gap is zero every time. But what about consumption euler equation? It turns out that to maintain expected and present output gap to be zero and to form an equilibrium, real interest rate should be set at natural rate of interest, similar to the previous case. (And this, without resorting to consumption euler equation, makes sense intuitively! As we are anchoring output to natural output, real interest rate also must be at natural level!) Are the problems underlined above are really problems? This would involve people’s subjective opinions, and as such, we will not go over this. Also please note that the descriptions above strictly only apply for basic New Keynesian model, but it is also equivalent important to note that some New Keynesian DSGE models do inherit these “problems.”

**News of future income**

Some have objected to New Keynesian consumption euler equation because it states that news of less income tomorrow raises today’s consumption. Consumption euler equation states that if real interest rate is lowered, difference between expected output at \( t + 1 \) and output at \( t \) becomes less, and this is indeed equivalent to saying that new of less income tomorrow raises today’s consumption. While these two are exactly the same thing, consequences seem very different, very similar to the jokes surrounding equivalence of axiom of choice, Zorn’s lemma and well-ordering theorem in axiomatic mathematics. Thinking in perspectives of mainstream economics, there is indeed possibility of reconciling contractionary effects of news that future incomes will be decreased and original consumption euler equation by adding a new parameter and re-work out utility functions and budget constraints. One may object to this, by saying that this is different from standard approaches of starting out from utility function and budget constraints. But it is important to note that New Keynesian economics itself was born out of discrepancy of early New Classical models. As with Calvo pricing, which has possibility of violating Lucas critique, sometimes things may have to be worked out backward if we are to work under mainstream paradigm. (For sure this does not mean that one has to work under mainstream paradigm.)
Do-Not-Take-Models-Seriously arguments These arguments made by some economists say that models are just models and if there are weird consequences, we can cherry-pick consequences that work nicely and use them. The problem with this argument is that this exactly justifies the use of old MIT Keynesian models that many mainstream economists object to. Of course Old Keynesians might be more right than New Keynesians. But if mainstream economists are to state that Old Keynesian models should not be taught except for historic (and educational connection) purposes, “models are just models - just cherry-pick good parts of the model” arguments should not be used.

Appendix

1. Stories of Marshallian firm theory

This is from the other chapter, though for fostering discussion, I present it here. The theory of perfect competition often employs the argument of infinite agents to say that perfect competition both satisfies profit maximization and demand equals supply. When following the mathematical logic, this is indeed justifiable, although the assumption of infinite number of agents seems very open to criticism, as there can definitely be butterfly effects, causing huge divergence from the original equilibrium point, thwarting the original intention of the perfect competition theory to provide at least empirical guidelines for the states of economy. This issue will never be resolved unless a specific tatonnement process is agreed and tested in reality.

As this is only brief summary, I intend to skip a few things, but for the purpose of summary, here is the following:

1. General equilibrium theory and partial perfect competition equilibrium are not really connected, and only the final equilibria formed are connected. If one believes that a supply curve for a single firm is needed for microeconomic analysis of markets, then partial perfect competition equilibrium is not just a one-market approximation to general equilibrium. This occurs, because the only way to justify demand curve and supply curve in partial equilibrium is by the original Cournot theorem that posits perfect competition market is derived from Cournot oligopoly. When this stance is adopted, single firm’s supply curve ceases to exist, and each firm is dependent on other firm’s choices. If single firm’s supply curve
does exist and assuming that firms maximize their profits, the equilibrium formed cannot be perfect competition equilibrium. The equilibria of general equilibrium and partial perfect competition do meet at the same point in some cases and I do not reject this. This convergence is in fact somehow coincidental features of the model. However, the justification used to justify partial equilibrium is not used in general equilibrium, and when the methods adopted in general equilibrium are used for partial equilibrium scenario, equilibria are guaranteed not to meet together. The only way to connect partial equilibrium and general equilibrium in methodological sense is by just assuming that every firm is a price taker, without any mathematical or logic explanation of why firms, inherently price setters, become price takers. Therefore, it is wrong to justify the assumption of auctioneer in general equilibrium by the recourse to partial equilibrium, or justify partial equilibrium based on general equilibrium. The below paragraph is the proof of what is said above.

Let us adopt the general equilibrium story of individual firms having separate demand and supply functions. If this is applied to the story of partial equilibrium, then, \( MR = P + q_i \frac{dP}{dq_i} \) where \( P \) is price function of \( q_i \) and by law of demand, this function is downward-sloping - I exclude the case of Giffen goods. I do not adopt that price function is a horizontal function, because it cannot be justified other than an ad-hoc way which is often claimed to be intuitive, but is not even intuitive at all. If one adopt horizontal price/demand function, which means that \( \frac{\partial P}{\partial q_i} = 0 \), which eventually leads to \( \frac{dP}{dq_i} = 0 \), meaning price is constant. This can only be justified if we say that price is pre-determined by an auctioneer and not by market force. While it is understandable that economic theorists think that firms may assume price first then set quantity, which can be justifiable, it must be remembered that markets are dynamic, not static. In this analysis, what happens if some firm/firms cut or increase their quantity? If price still stayed the same, then we are violating law of demand at the market level. If price increased, then horizontal price functions that individual firms have cannot be kept. Therefore, I assume that market demand function has the same slope as single firm’s demand function, assuming \( nq_i = Q \) by symmetry of firms. As firms do not produce zero quantity, but positive quantity, whether infinitesimal or not, \( MR \), marginal revenue, would be smaller than price. By profit-maximization, \( P + q_i \frac{dP}{dq_i} = MC \), and according to the original Cournot theorem, perfect competition is derived because \( q_i \to 0 \) as \( n \to \infty \), which is \( P = MC \). But if we assume that individual firms do have their own supply and demand function that respect their own law of demand, as in general equilibrium, while \( P = MC \) is approached as \( n \to \infty \), the deviation that each firm makes from \( P = MC \)
build up until deviation is no longer infinitesimal. To prove this argument, suppose that price and supply function is a linear function of quantity. 

$$P_i = -kq_i + a, MC_i = cq_i + d$$ and $$MR_i = -2kq_i + a$$ at the individual firm level. Profit-maximizing point is then at $$q_i = (a - d)/(2k + c)$$. If $$P = MC$$ is hold, then $$q_i$$ is instead $$(a - d)/(k + c)$$. The difference between these two $$q_i$$ is $$\Delta q_i = k(a - d)/[(k + c)(2k + c)]$$, and this deviation at the market level is $$\Delta Q = nk(a - d)/[(k + c)(2k + c)]$$. $$k$$ is assumed to be positive real number, meaning that denominator will be more than infinitesimal hyperreal. $$n(a - d)$$ needs to be more than infinitesimal hyperreal, as market quantity $$Q$$ is assumed to be non-infinitesimal. Therefore, market quantity/price deviation would be non-infinitesimal.

2. The first argument is not very simple to understand, and cannot be completely covered by just logical arguments here. It is quite easy to think that "as equilibria points converge, everything is OK" when it is not. But for time issues, I will hand this to a separate part of the whole writing, here called "the other chapter." But the second argument is more clear. Look at supply curve of the market. As the supply quantity in the market is determined by the sum of each firm’s supply, assuming that every firm produces the same amount $$q_i$$, the market quantity would be $$nq_i$$, where $$n$$ is the number of firms and can be assumed to be a hyperreal number, when the number of firms is infinite. Let us assume a linear function for marginal cost curve of a single firm, $$MC(q_i) = kq_i + c$$, where $$k$$ is some positive real number (one can instead use hyperreal number for the case of handling infinity) and $$c$$ has the same condition except that this number cannot be infinite. This means that the supply curve of market would be:

$$S(Q) = (k/n)Q + c$$

, where $$Q = nq_i$$. If we are to keep $$k/n$$ to be a positive integer, this means that $$k \to \infty$$ as $$n \to \infty$$. This means that for even producing some small quantity, $$q_i$$, marginal cost explodes for every firm. (Let us think about the case where a firm produces 0.01 quantity. Then if $$n \to \infty$$, marginal cost would be approaching $$\infty$$. While this may be OK mathematically, this is completely out of reality and cannot be accepted. The point that firms are producing closed to zero amount has no relevance here. As this cannot occur, the only way to remedy this is by assuming that $$k$$ is non-infinite value and therefore, $$k/n$$ approaches 0. This suggests that when economic equations of supply/demand in perfect competition theory are written in a correct way, even when accepting Cournot oligopoly derivation, supply curve becomes converges to a flat supply curve, as the number of firms begins to increase. This is very different from what neoclassicals often
portray in standard textbooks, and in this "corrected" model, roles of supply and demand become somehow different. Whether this model can be accepted or not is a different issue and I will not discuss it, and I myself am opposed to the adoption of a such model. Surprisingly, what this model suggests seems to share some features of Kalecki’s theory of firms and some Post-Keynesian literature, although this is not a good derivation. First of all, importance of exogenous cost shocks become very important in this model, as it directly impacts price level. Secondly, demand increases do not directly change price increases. Therefore, economy can be thought of as demand-driven, as many Keynesians, regardless of New or Post, claim. Supply can only have marginal effects except when some shocks have occurred, pushing supply curve upward. Extensions to non-linear cases can be done easily, and the results are basically the same assuming upward-sloping marginal cost curve. Further discussion will be provided in a separate chapter of the whole writing.

3. This third argument is that horizontal demand curve is unjustified regardless of whether the number of firms is assumed to be infinite or not. I did provide a simplified proof above, but the full deviation of the all three points will be dealt in a separate writing.

2. How not to confuse partial derivative relationships

In Keen (2010), Steve Keen and Russell Standish claimed that they "disproved" Marshallian firm theory (perfect competition) mathematically. The argument they used is that as firms do not consider quantity of other firms when profit-maximizing in Marshallian firm theory, one can set \( \partial q_i / \partial q_j = 0 \) where \( i \neq j \), and \( i \) and \( j \) represents each firm and \( q_i \) is the quantity firm \( i \) produces. But this proof unintentionally makes \( Q \), market quantity, constant, and this is definitely not the formulation of Marshallian firm theory. It is true that firms consider profit function \( \Pi \) and tries to find out \( q_i \) that \( \partial \Pi / \partial q_i = 0 \) (first-order condition), this itself does not make \( \partial q_i / \partial q_j = 0 \) where \( i \neq j \). This erroneous assumption of \( \partial q_i / \partial q_j = 0 \) leads the authors to conclude that they found mathematical flaws in profit-maximizing function. \( \partial q_i / \partial q_j \) can be justified if the authors started from horizontal demand curve of a single firm, but the authors first start by rejecting horizontal demand curve. (If every firm has horizontal demand function, there is not really a reason for other firms to change their quantity, so \( \partial q_i / \partial q_j = 0 \) might be justifiable, but this assumption is not even needed and may lead to a wrong conclusion. It is best to avoid this assumption.)
3. Neoclassical Case Example 1

When housing supply is in excess, government decides to cut its own supply provided to housing markets so that price may increase. By Marshallian cross, market is always in equilibrium. We do not have any consensus on what happens if disequilibrium and failure of a market to clear last for noticeable long-term periods. Government says that it justifies its supply cut by law of supply and demand - when supply decreases, price increases. And one goes on to read textbooks and find no justification for it....

4. Neoclassical Case Example 2

There is a person who takes neoclassical doctrines too seriously that he concludes something that even many pure neoclassical economists and pure New Classical economists do not accept. He lives in a fictional world, where labour unions and protests are illegal. Labour supply curve is upward-sloping, as all supply curves are. He knows that for his purpose, backward-bending supply curve of labour does not matter. One day, for no reason, every firm decides to cut wages. After hearing the news, he concludes that many people would go on a holiday.

5. Philosophical issues with solving social problems by economic methods

Suppose that there is a group of people who smoke and the other group of people who do not smoke. The group of people who do not smoke complain that their utility is less because of the people who smoke. Let us say that the number of people who smoke is greater than the number of people who do not smoke. In this case, given that the group of the people who smoke gain utility by smoking, what would be the right method to solve this problem? Neoclassical economist comes in and says that things are simple. Minimize and eliminate the transaction cost that prevents people from negotiating, and proper equilibrium can form. However, as people who do not smoke are in majority, they reject the neoclassical resolution and take a vote in a parliament to ban smoking. The question comes: is this immoral? The answer is of course controversial, and it is obvious that many people will disagree that neoclassical solution is the best solution, regardless of whether equilibrium holds or not.
References


Rowe, N. “New Keynesians Just Assume Full Employment Without Even Realising it.” (website)

Rowe, N. “New Keynesian Countercyclical Fiscal Policy Isn’t What You Probably Think It Is.” (website)